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SCATTERING OF A LONGITUDINAL WAVE BY A SPHERICAL CAVITY WITH A FLUID IN AN ELASTIC POROUS SATURATED MEDIUM*

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The diffraction of a plane longitudinal harmonic wave propagating in an unbounded elastic porous permeable fluid-saturated medium, by a spherical cavity with a fluid is examined. The Frenkel-Biot model /1—3/ is used to describe the dynamics of the porous saturated elastic medium.

The equations describing the space of harmonic waves in a saturated porous medium have the form

$$\omega^2\rho_{11}\mathbf{U} + \omega^2\rho_{12}\mathbf{V} = i\omega b\,(\mathbf{V} - \mathbf{U}) - N\Delta\mathbf{U} - \nabla\,[(A + N)\,\nabla\mathbf{U} + Q\nabla\mathbf{V}],\quad b = \eta\Phi^2F\,(\omega)/K \tag{1}$$
$$\omega^2\rho_{12}\mathbf{U} + \omega^2\rho_{22}\mathbf{V} = i\omega b\,(\mathbf{U} - \mathbf{V}) - \nabla\,(Q\nabla\mathbf{U} + R\nabla\mathbf{V})$$

Here $\mathbf{U}$ and $\mathbf{V}$ are, respectively, the displacement vectors of the solid and liquid phases in the pores, ρ_{11} and ρ_{22} are the coefficients of dynamic density, ρ_{12} is the mass coupling coefficient between the liquid and solid phases, A, N, Q, R are elastic constants, η is the dynamic viscosity of the fluid, Φ is the bulk porosity, K is the permeability, and ω is the angular frequency; the function $F(\omega)$ describes the deviation in the pores from Poiseuille flow /2/.

We represent the displacement vectors of the solid and liquid phases as the sum of displacement vectros in the incident and scattered waves, i.e.,

$$\mathbf{U} = \mathbf{\Pi} + \mathbf{u},\quad \mathbf{V} = \boldsymbol{\pi} + \mathbf{v}$$

A plane travelling wave with the displacements $\mathbf{\Pi}$ in the solid and $\boldsymbol{\pi}$ in the liquid phases satisfies system (1). Therefore, because of the linearity of this system, the fields $\mathbf{u}$ and $\mathbf{v}$ of the scattered waves satisfy them. Introducing the spherical coordinates r, θ, φ with origin at the centre of a spherical cavity of radius a and polar axis coincident with the direction of incident wave propagation, we determine the complex amplitudes of the scattered wave displacement potentials by the following relationships:

$$\mathbf{u} = \nabla\Lambda + \nabla \times (\Psi\mathbf{e}_\varphi),\quad \mathbf{v} = \nabla B + \nabla \times (\chi\mathbf{e}_\varphi) \tag{2}$$

where $\mathbf{e}_\varphi$ is the unit vector in the equatorial direction, and the factor $\exp(-i\omega t)$ is omitted everywhere.

Substituting (2) into (1) applying the divergence operation to the system of equations, we reduce it to the form

$$q_{1k}\Delta\Lambda + q_{k2}\Delta B + \varkappa^2(\gamma_{1k}\Lambda + \gamma_{k2}B) = 0;\quad k = 1, 2$$
$$q_{11} = A + 2N,\quad q_{12} = Q/H,\quad q_{22} = R/H,\quad H = A + 2N + 2Q + R$$
$$\varkappa^2 = \omega^2\rho/H,\quad \rho = \rho_1(1 - \Phi) + \rho_2\Phi,\quad \gamma_{11} = (\rho_{11} + ib/\omega)/\rho$$
$$\gamma_{12} = (\rho_{12} - ib/\omega)/\rho,\quad \gamma_{22} = (\rho_{22} + ib/\omega)/\rho$$

where ρ_1 and ρ_2 are the densities of the solid and liquid phase material in the pores.

Performing the substitution $\Lambda = \Lambda_1 + \Lambda_2$, $B = m_1\Lambda_1 + m_2\Lambda_2$, we require that Λ_1 and Λ_2 satisfy the Helmholtz equations

$$\Delta\Lambda_i + \varkappa^2\xi_i\,\Lambda_i = 0 \tag{3}$$

For this it is necessary that ξ_1 and ξ_2 should be the roots of the following dispersion equation:

$$(q_{11}q_{22} - q_{12}^2)\,\xi^2 - (\gamma_{11}q_{22} + \gamma_{22}q_{11} - 2\gamma_{12}q_{12})\,\xi + (\gamma_{11}\gamma_{22} - \gamma_{12}^2) = 0 \tag{4}$$

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Then $$m_i = (\gamma_{12} - \xi_i q_{12})/(\xi_i q_{22} - \gamma_{12})$$

Substituting (20) into (1) and applying the vortex operation to the system, we obtain, apart from a term in the form of a harmonic function,

$$\chi = -(\gamma_{12}/\gamma_{22})\psi, \quad k_3^2 = \frac{\varkappa^2 H}{N\gamma_{22}}(\gamma_{11}\gamma_{22} - \gamma_{12}^2) \tag{5}$$

$$\left(\Delta - \frac{1}{r^2 \sin\theta} + k_3^2\right)\psi = 0 \tag{6}$$

The dispersion equation (4) is obtained in /5/. Rather awkward formulas for ξ_i and m_i are also given there, which we do not write down here. The least wave number k_1 in absolute value corresponds to a longitudinal wave of the first kind.

The scalar potential L of the displacement w in the fluid, defined by the relationship $\mathbf{w} = \nabla L$ satisfies the Helmholtz equation

$$\Delta L + k^2 L = 0 \tag{7}$$

where k is the wave number for plane longitudinal waves in the fluid.

The solutions of (3), (6) and (7) have the form /4/

$$\Lambda_i = \sum_{n=0}^{\infty} \alpha_n^{(i)} h_n^{(1)}(k_i r) P_n(\cos\theta), \quad r \geqslant a \tag{8}$$

$$\psi = \sum_{n=0}^{\infty} \beta_n h_n^{(1)}(k_3 r) P_n^{(1)}(\cos\theta), \quad r \geqslant a; \quad L = \sum_{n=0}^{\infty} \alpha_n j_n(kr) P_n(\cos\theta), \quad r \leqslant a$$

Here $h_n^{(1)}(x)$ and $j_n(x)$ are Hankel spherical functions of the first kind and Bessel spherical functions of order n and argument x, while $P_n(x)$ and $P_n^{(1)}(x)$ are Legendre functions of a real argument.

An incident plane longitudinal wave of the first kind can also be expanded in a series of spherical functions /4/

$$\exp(ik_1 r\cos\theta) = \sum_{n=0}^{\infty} i^n (2n+1) j_n(k_1 r) P_n(\cos\theta) \tag{9}$$

The following conditions must be satisfied on the boundary $r = a$ between the porous medium and the fluid /5/:

$$(1 - \Phi) U_r + \Phi V_r = w_r \tag{10}$$

$$\Gamma_{rr} = -p_L, \quad p = p_L, \quad \Gamma_{r\theta} = 0 \tag{11}$$

where w_r is the radial component of the displacement of the fluid filling the cavity, Γ_{ij} are the total stress tensor components in the porous medium, and p, p_L are, respectively, the fluid pressure in the pores and in the cavity.

Condition (10) expresses the continuity of the normal displacements on the interface, the first two relationships in (11), the continuity of the total stress and pressure components normal to the boundary, and the last relationship in (11), the condition for there to be no tangential stresses on the boundary with the fluid.

The amplitudes of the scattered and excited waves are found by solving the system of linear equations obtained after substituting expansions (8) and (9) for the scattered and incident wave potentials into the boundary conditions (10) and (11).

The incident wave energy flux density and the scattered wave energy fluxes are determined by the expressions

$$j_1 = -\frac{\omega}{2}[A + 2N + 2Q\,\mathrm{Re}\,m_1 + R\,|m_1|^2]\,|k_1|^2\,\mathrm{Re}\,k_1 \tag{12}$$

$$J_i = -2\pi\omega \exp[-2\,\mathrm{Im}(k_i a)][(A + 2N + 2Q\,\mathrm{Re}\,m_i +$$

$$R\,|m_i|^2)]\,\mathrm{Re}\,k_i \sum_{n=0}^{\infty} \frac{1}{2n+1}|\alpha_n^{(i)}|^2 \quad (i = 1, 2)$$

$$J_3 = -2\pi\omega N\,\mathrm{Re}\,k_3 \exp[-2\,\mathrm{Im}(k_3 a)] \sum_{n=1}^{\infty} \frac{n(n+1)}{2n+1}|\beta_n|^2$$

Here j_1 is the incident wave energy denisty, J_i are the scattered longitudinal wave energy flux of the i-th kind, and J_3 is the energy flux excited because of transverse wave scattering. The scattering cross-sections are defined as the ratios of the scattered wave energy fluxes to the incident wave energy flux density.

Out-of-phase motion of the solid and liquid phase particles corresponds approximately to a longitudinal wave of the second kind, hence its damping is large, and as a rule, it is not recorded in experiments on the acoustics of consolidated porous media. Consequently, the computations were performed just for an incident longitudinal wave of the first kind. The scattering cross-sections by a spherical cavity with fluid in an equivalent single-phase elastic medium were also computed for comparison. The equivalence was understood in the sense of equal mean densities and agreement between the longitudinal and transverse wave velocities in the limit case as $\omega \to 0$.

The parameters of the fluid filling the cavity and the pores of the surrounding medium corresponded to water under normal conditions. The following quantities were given for the porous medium: porosity $\Phi = 0.25$, limit values of the longitudinal wave velocity $c_p = 3800$ m/sec and transverse wave velocity $c_s = 2200$ m/sec as $\omega \to 0$ and mean density $\rho = 2.35 \cdot 10^3$ kg/m^3. The solid phase material parameters corresponded to limestone under natural conditions. The magnitude of the mass coupling coefficient between the phases was selected to be /6/ $\rho_{12} = -{}^2/_3\Phi(1-\Phi)\rho_2 \approx -0.125\rho_2$.

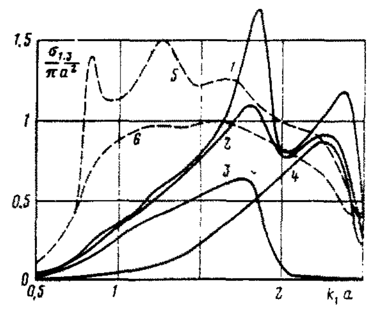

Fig.1

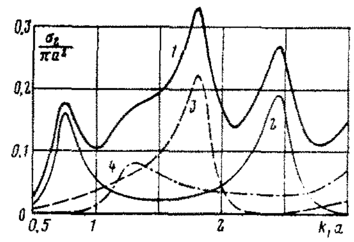

Fig.2

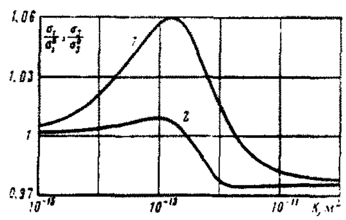

Fig.3

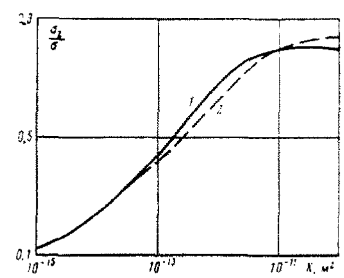

Fig.4

The results of computing the scattering cross-section σ_1 for a longitudinal wave of the first kind are represented in Fig.1 by the continuous lines. Curve *1* represents the dependence of the longitudinal wave scattering cross-section in a single-phase elastic medium by a spherical cavity with fluid in the frequency, and curve *2* is the frequency dependence of the scattering cross-section of a longitudinal wave of the first kind in a porous medium. In both cases the scattering sections are resonant in nature. The first two resonances are presented on the graph, where the centrally symmetric radial vibrations specify the fundamental mode, and the second the dipole mode. Scattering cross-sections specified by these modes are given (curves *3* and *4*).

A singificant decrease in the scattering cross-section is observed for a 10^{-13} m^2 permeability of the prous medium as compared with an equivalent single-phase medium, especially in the domain of the principal resonance. This diminution is associated with the drop in the quality of the resonances because of the losses in the radiation of the longitudinal wave of the second kind. Moreover, the resonances are shifted somewhat towards the low-frequency side since an additional apparent mass of fluid vibrating in the pores occurs.

The transverse wave excitation cross-section σ_3 in the same frequency range (the dashed lines in Fig.1) has several maxima in a single-phase medium (curve *5*), which are smoothed out for the two-phase medium (curve *6*).

The excitation cross-section of a longitudinal wave of the second kind also has a resonant nature (Fig.2, curve *1*). The lowest-frequency resonance scattering occurs because of dipole scattering (curve *2*), here the excitation cross-section of a longitudinal wave of the second kind exceeds the scattering cross-section of a longitudinal wave of the first kind. Monopole (curve *3*) and quadrupole (curve *4*) scatterings affect the frequency dependence of the excitation section of a longitudinal wave of the second kind substantially (curve *1*). From a comparison of Figs.1 and 2 it follows that the resonance excitations of scattered longitudinal waves of the second kind are due to resonance excitations of both the scattered longitudinal waves of the first kind and the scattered transverse waves. The latter is explained by the fact that the cavity wall motion is of maximum amplitude in the resonance domain of these kinds of waves.

In the low-frequency domain $|k_1a|$ and $|k_3a|$ are small compared to $|k_2a|$, which results in a large contribution of effects associated with waves of the second kind to the magnitude of the total scattering cross-sections for fairly high permeability. The dependences on the permeability K of the ratio between the scattering cross-section of a longitudinal wave of the first kind σ_1 and the excitation cross-section of a transverse wave σ_3 in a porous medium and the corresponding sections σ_1° and σ_3° in an equivalent single-phase medium for $|k_1a| = 0.16$ are given in Fig.3. As the permeability decreases, both ratios σ_1/σ_1° (curve *1*) and σ_3/σ_3° (curve *2*) tend to one since the medium goes over into a single-phase medium as $K \to 0$. In the domain of high permeabilities the sections cease to depend on K because of the smallness of the influence of the viscous effects $(b = 0)$.

The ratio of the excitation cross-section of the scattered longitudinal wave of the second kind σ_2 to the total scattering cross-section of all kinds of waves σ grows rapidly as the permeability increases (Fig.4) and for $K = 10^{-12} - 10^{-10}$ m^2 reaches a value of 0.7–0.8. The dependences presented were computed for $\rho_{12} = 0$ (curve *1*) and $\rho_{12} = -0.125\rho_2$ (curve *2*). At low frequencies the scattering cross-sections of the longitudinal wave of the first kind and the cross-section of transverse wave formation depend weakly on the permeability of the saturated porous medium (Fig.3). In contrast, the excitation cross-section of the wave of the second kind depends strongly on the permeability, and even exceeds the scattering cross-section of the wave of the first kind for permeabilities of $10^{-14} - 10^{-13}$ m^2. For $K > 2.7\cdot 10^{-13}$ m^2 the excitation cross-section of a longitudinal wave of the second kind exceeds the total scattering cross-section of the longitudinal wave of the second kind and the transverse wave, i.e., $(\sigma_2/\sigma) > 0.5$ (Fig.4).

Therefore, the main distinctions of wave scattering by cavities with fluid in a saturated porous and in an elastic single-phase medium at both resonance scattering and scattering at low frequencies are due to the formation of waves of the second kind, i.e., to hydrodynamic effects near the cavity boundaries.

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